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ON THE ELECTRIC CONDUCTIVITY OF A SUSPENSION OF HOMOGENEOUS ELLIPSOIDS OF REVOLUTION

WITH SPECIAL REFERENCE TO AN ORIENTATION EFFECT

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INTRODUCTION

The electrical conductivity of a suspension is dependent partly upon the direct conduction through the suspending medium and the suspended particles, partly upon the cataphoretic movement of the particles. The latter phenomenon is caused by the setting-up of an electric double-layer between the suspended particles and the suspending medium. On establishment of an electric field the particles move and transport charges of electricity; the liquid layer, taking the opposite charge, also moves. The corresponding contribution to the conductivity of the system depends upon the effective charges, the number and frictional resistance of the solid particles, as well as upon the charges of the moving liquid layer.

The present paper does not propose to deal with particular aspects of this cataphoretic phenomenon but exclusively with the question as to how the aforementioned direct conduction can be influenced by the orientation of particles of non-spherical form suspended in the medium.

The suspended particles are assumed to be solid conductive ellipsoids, and the problem with which we are here concerned is how the conductivity of the suspension depends upon the orientation of the ellipsoid particles.

Let the liquid medium have a dielectric constant ε_0 and an electric conductivity σ_0, and let the suspended inelastic ("starr-elastische") rotation-ellipsoidal particles have a dielectric constant ε and a conductivity σ.

Provided that the conductivity of the ellipsoids is different from that of the suspending medium, the conductivity of the suspension will depend upon the orientation of the particles, assuming one extreme value (maximum) if all the ellipsoids are adjusted with their longitudinal axes parallel to the direction of the current, and the other extreme value (minimum) if they are orientated altogether irregularly.

When $\varepsilon_0 \neq \varepsilon$ or $\sigma_0 \neq \sigma$ or both, ponderomotive forces will act on the ellipsoids, which will thereby become subject to a moment of rotation and tend to attain a certain orientation.

The problem will then be to follow more closely this process of orientation during the application of an electric field, and the resulting changes in the conductivity.

We have therefore to consider the electric rotation moment, the opposing moment of friction arising from the rotation of ellipsoids, and the influence of the Brownian movements upon an incipient orientation, and then calculate the conductivity of the suspension for a given orientation of ellipsoids.

I. The Electric Rotation Moment.

Suppose that the electrodes of the conductivity vessel are two vertical plates, only slightly apart in proportion to their extent.

If there are no ellipsoids between the electrodes, the field is homogeneous with horizontal lines of force.

If an ellipsoid is placed between the electrodes, the field will be deformed locally, and the ellipsoid will be affected by a rotation moment D.

We place a rectangular right-coordinate system in the ellipsoid, with the initial point in its centre and the x-axis coincident with the axis of symmetry (the short half-axis for planetary (oblate) ellipsoids of revolution, the long half-axis for prolate ellipsoids).

Since we are considering an ellipsoid of revolution, we may choose the z-axis at right angles to the plane formed by the x-axis and the outer given field-vector, strength of field $\underline{E}$, so that $E_z = 0$. This gives the y-axis.

The acute angle between the symmetrical axis (x-axis) and the field vector is called ϑ.

The moment of revolution exerted by the field upon the ellipsoid was calculated by R. FÜRTH[1 2]. Retaining his terms we get:

$$D = \frac{2\,\varepsilon_0\,E^2 \sin 2\,\vartheta}{\left(X_0 + \dfrac{4}{\mu - 1}\right)\left(Y_0 + \dfrac{4}{\mu - 1}\right)}\;\frac{b^2}{a^2}\,(a^2 - b^2)\,\lambda, \text{ where}$$

$$\lambda = \int_{-a}^{+a} \frac{x^2\,(a^2 - x^2)}{x^2\left(1 - \dfrac{b^2}{a^2}\right) - a^2}\,dx, \qquad \mu = \frac{\sigma}{\sigma_0}$$

and X_0 and Y_0 are constants.

The moment of revolution is positive and tends to increase ϑ when $a < b$, and negative, tending to decrease ϑ, when $a > b$.

For oblate ellipsoids of revolution we have:

[1] R. FÜRTH: Zeit. f. Physik. **22**: 98, 1924.
[2] R. FÜRTH: Zeit. f. Physik. **44**: 256, 1927.

$$\frac{b}{a} = \varkappa > 1;\ \lambda = \frac{-2a^3}{(\varkappa^2-1)^2}\left(\frac{1}{3} - 2\sqrt{\varkappa^2-1}\ \mathrm{arc\ tg}\ \sqrt{\varkappa^2-1} + \frac{2}{3}\varkappa^2\right)$$

$$X_0 = \frac{4\varkappa^2}{\varkappa^2-1}\left[1 - \frac{\pi}{2\sqrt{\varkappa^2-1}} + \frac{1}{\sqrt{\varkappa^2-1}}\ \mathrm{arc\ tg}\ \frac{1}{\sqrt{\varkappa^2-1}}\right]$$

$$Y_0 = \frac{2\varkappa^2}{\varkappa^2-1}\left[-\frac{1}{\varkappa^2} + \frac{\pi}{2\sqrt{\varkappa^2-1}} - \frac{1}{\sqrt{\varkappa^2-1}}\ \mathrm{arc\ tg}\ \frac{1}{\sqrt{\varkappa^2-1}}\right].$$

For prolate ellipsoids of revolution:

$$\frac{b}{a} = \varkappa < 1;\ \lambda = \frac{-2a^3}{(1-\varkappa^2)^2}\left(\frac{1}{3} + \frac{2\varkappa^2}{3} + \frac{\varkappa^2}{2\sqrt{1-\varkappa^2}}\log\frac{1-\sqrt{1-\varkappa^2}}{1+\sqrt{1-\varkappa^2}}\right)$$

$$X_0 = \frac{4\varkappa^2}{1-\varkappa^2}\left[-1 + \frac{1}{2\sqrt{1-\varkappa^2}}\log\frac{1+\sqrt{1-\varkappa^2}}{1-\sqrt{1-\varkappa^2}}\right]$$

$$Y_0 = \frac{\varkappa^2}{1-\varkappa^2}\left[\frac{2}{\varkappa^2} - \frac{1}{\sqrt{1-\varkappa^2}}\log\frac{1+\sqrt{1-\varkappa^2}}{1-\sqrt{1-\varkappa^2}}\right].$$

We see that the expression for D is not dependent upon the ellipsoidal dielectric constant ε, owing to the fact that the electric field inside the ellipsoid is homogeneous.

The moment of revolution is proportional to the square of the field strength, so that D is independent of the direction of the field.

As stated by Fürth, the expression for D applies to quasistationary conditions only (cf. his discussion with Busch[1]).

As the moment of revolution results essentially from accumulation of electricity on the surface of the ellipsoid, owing to the difference in the conductivity of the ellipsoid and the suspension medium, we have $D = 0$ for $\sigma_0 = \sigma$.

On account of D's proportionality to $\sin 2\vartheta$, the moment of revolution will have a maximum for $2\vartheta = \frac{\pi}{2}$ or $\vartheta = 45°$.

[1] R. Fürth: Phys. Zeit. **25**: 679, 1924.

With $0 < \vartheta < \frac{\pi}{2}$, D has a definite sign, positive for oblate, negative for prolate ellipsoids.

If the symmetrical axis is parallel to the direction of the lines of force, or is at right angles to these lines, we have $D = 0$.

X_0, Y_0, a, b and λ depend only on the geometrical form of the ellipsoid; with $a = b$, we have $D = 0$.

So, independently of the magnitude of its electric constants ($\varepsilon \gtrless \varepsilon_0$, $\sigma \gtrless \sigma_0$) the ellipsoid will tend to adjust itself with its longitudinal axis parallel to the field.

In the present calculations no consideration has been given to the phenomenon of cataphoresis.

The conductivity of a suspension is increased on account of the cataphoresis of the particles (Smoluchowski[1]), as the slip in the Helmholtz double layer causes a transfer of electric charges.

For a suspension of spherical particles the cataphoretic conductivity is

$$\lambda = \frac{4\,\pi\nu\eta\, r\,(r+\delta)\,u^2}{N\,\delta}$$

where λ is the specific conductivity, ν the number of particles per c. c., η the coefficient of inner friction, δ the thickness of the double layer, r the radius of the particles, u the cataphoretic mobility (cm/sec./volt/cm), N the Avogadro-Loschmidt number.

Whether the cataphoretic conductivity should be allowed for in measuring the conductivity of suspensions, has been

[1] M. Smoluchowski: Anzeig. d. Akad. d. Wiss. Krakau. 1903, p. 185. Phys. Zeit. **6**: 529, 1905.

discussed by Freundlich[1] and de Hevesy[2]. The increase of conductivity is constant, however, and has no influence on the orientation effect, as the symmetrical distribution of the charges in the double layer cannot produce any rotation of the ellipsoid.

There is no orientation of the particles in the direction of least hydrodynamic resistance, brought about by cataphoresis, electro-endosmotic flow or translational Brownian movements, as in this sense there is no minimal principle of liquid resistance (R. Gans[3] and C. E. Marshall[4]).

In narrow vessels, however, the different velocities of the liquid layers will be able to cause an orientation (cf. orientation of particles by "streaming double refraction").

The velocity of cataphoretic movement is low; according to Helmholtz-Smoluchowski's theory[5] and Freundlich and Abramson's experiments[6] it is independent of the form of the particles, that is, independent of the orientation of the ellipsoids.

II. The Hydrodynamic Problem.

The moment of friction for an ellipsoid rotating about a main axis in a viscous medium was first calculated by Edwardes[7].

[1] H. Freundlich: Kapillarchemie. Akadem. Verlagsg. vol. I, p. 351, 1930. vol. II, p. 63, 1932.

[2] G. de Hevesy: Koll. Zeitschr. **21**: 136, 1917.

[3] R. Gans: Sitz. br. d. math.-phys. Klasse d. K. B. Akadem. d. Wiss. zu München, 1911, p. 191.

[4] G. E. Marshall: Transact. of the Faraday Soc. **26**: 173, 1930.

[5] L. Graetz: Handbuch der Elektrizität und d. Magnetismus. Ambr. Barth, Leipzig. vol. II, p. 381, 1912—21.

[6] H. Freundlich and H. A. Abramson: Zeit. f. phys. Chemie. **133**: 51, 1928.

[7] Edwardes: Quart. Journ. Math. **26**: 70, 1892.

The coefficient of resistance ξ for rotation of an ellipsoid of revolution about an axis at right angles to the symmetrical axis is

$$\xi = \frac{16\pi\eta}{3} \frac{a^2 + b^2}{a^2 A + b^2 B}$$

where a og b are the lengths of the semi-axes, η the coefficient of the internal friction of the medium, and A and B are constants expressed by

$$A = \int_0^\infty \frac{ds}{(a^2+s)(b^2+s)\sqrt{a^2+s}} = \frac{X_0}{2ab^2}$$

$$B = \int_0^\infty \frac{ds}{(b^2+s)^2\sqrt{a^2+s}} = \frac{Y_0{}^2}{2ab^2}$$

X_0 and Y_0 are the integrals given in Section I.

Edwardes' concluding formula is incorrect, as the numerical factor $^{32}/_5$ must be replaced by $^{16}/_3$ when the coefficient of the last term of the preceding equation is corrected to 4.

III. Calculation of a Distributive Function for Orientation of the Ellipsoids in Relation to Time.

If a polar coordinate system (ϑ, φ) is introduced into the sphere of unit with the direction of the field for polar axis, and if from the initial point we draw for each ellipsoid a radius vector parallel to the momentary direction of the symmetrical axis, the density of intersections on the surface of the sphere will, for symmetrical reasons, statistically, depend only on the polar angle ϑ, the angle between the field vector and the symmetrical axis, and on the time, but not on φ.

Giving the term $fd\Omega$ to the relative number of ellipsoids with the symmetrical axis within a solid angle $d\Omega$ around the direction (ϑ, φ), the distributive function $f(t, \vartheta)$ will be altered, partly due to the electric field which causes the orientation of the ellipsoids, partly due to the Brownian rotation movement.

This alteration of f can be calculated by a method analogous to that employed by Debye[1] for calculation of the orientation of polar molecules in electric fields.

Instead of the simple Maxwell-Boltzmann distribution function, which applies to statistical equilibrium only P. Debye—on the basis of Einstein's theory concerning the Brownian movement—derived a partial differential equation determining the distribution function for orientation of polar molecules.

Applied to the present case, this method will only be roughly outlined. The number of ellipsoids, the direction of whose symmetrical axes in the time interval δt runs more into than out of $d\Omega$, is given by the equation

$$\delta t \frac{\partial f}{\partial t} d\Omega = \Delta_1 + \Delta_2$$

where Δ_1 signifies the part arising from the orientation effect of the field, Δ_2 the part due to the Brownian rotation movements.

If Θ is the angle between the axes of $d\Omega$ and an adjacent solid angle $d\Omega'$, the following expression applies:

$$\Delta_2 = d\Omega \frac{\overline{\Theta^2}}{4} \left[\frac{\cos\vartheta}{\sin\vartheta} \frac{\partial f}{\partial \vartheta} + \frac{\partial^2 f}{\partial \vartheta^2} \right].$$

From this expression it is evident that the Brownian rotation

[3] P. Debye: Polare Molekeln. Hirzel, Leipzig. 1929, p. 88.

movements have no influence if f is independent of ϑ, *i. e.*, in an altogether irregular distribution. As soon as the electric field induces an orientation (f dependent on ϑ), the Brownian rotation movements influence the distribution.

The rotation moment D, acting on the individual ellipsoid (mentioned in Section I) and causing it to rotate round an axis at right angles to the direction of the symmetrical axis and the field, tends to alter ϑ.

Under the influence of a constant rotation moment, the friction would make an ellipsoid rotate with a constant angular velocity, determined by the equation

$$D = \xi \frac{d\vartheta}{dt}$$

where ξ is the coefficient of resistance mentioned in Section II.

If the rotation moment is positive, the angular velocity is also positive, and vice versa.

Applying this equation to our case with a variable rotation moment, we shall again assume that the acceleration effect is negligible.

For Δ_1 we then have:

$$\Delta_1 = -\frac{\partial}{\partial\vartheta}\left(2\pi f \frac{D}{\xi}\delta t \sin\vartheta\right) d\vartheta .$$

Introducing Δ_1 and Δ_2 and $d\Omega = 2\pi \sin\vartheta \, d\vartheta$ in the original equation we obtain

$$\frac{\partial f}{\partial t} = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\left[\frac{\overline{\Theta^2}}{4\delta t}\frac{\partial f}{\partial\vartheta} - \frac{D}{\xi} f\right]\right).$$

For determination of the characteristic constant $\frac{\overline{\Theta^2}}{\delta t}$ we make use of the fact that the Maxwell-Boltzmann expression

$$f = A \cdot e^{-\frac{u}{kT}}$$

where u is the potential energy, k the Boltzmann constant (gas constant per single molecule), and T the absolute temperature, in the special case of $\frac{\partial f}{\partial t} = 0$ (constant field) must be a solution of the differential equation.

As in the stationary case

$$D = -\frac{\partial u}{\partial \vartheta}$$

it can be shown that the Maxwell-Boltzmann expression is a solution when:

$$\frac{\overline{\Theta^2}}{4\delta t} = \frac{kT}{\xi}.$$

Introduction of this expression gives the final differential equation for determination of the distribution function $f(t, \vartheta)$, which can then be found for an arbitrary period of the regulating moment of revolution.

$$\xi \frac{\partial f}{\partial t} = \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \left[\sin \vartheta \left(kT \frac{\partial f}{\partial \vartheta} - Df \right) \right]. \tag{I}$$

Solution of the Differential Equation and Determination of the Relaxation Time of the Particles.

We divide equation (I) by kT and transfer the term with $\frac{\partial f}{\partial \vartheta}$ to the left side. If then we express the complicated formula for the rotation moment (mentioned in Section I) as

$$D = D_0 \sin 2\vartheta$$

and put $\tau_0 = \dfrac{\xi}{kT}$ and $\alpha = \dfrac{2\,D_0}{kT}$

where α may be a function of the time, the differential equation for our problem is as follows:

$$(1)\quad \tau_0 \frac{\partial f}{\partial t} - \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left[\sin\vartheta\,\frac{\partial f}{\partial\vartheta}\right] = -\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left[\sin\vartheta\,\sin 2\,\vartheta\,\frac{\alpha}{2}\,f\right].$$

We first consider the homogeneous equation resulting from (1) by putting $\alpha = 0$:

$$(2)\qquad \tau_0 \frac{\partial f}{\partial t} - \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left[\sin\,\vartheta\,\frac{\partial f}{\partial\vartheta}\right] = 0.$$

Putting in the usual way,

$$f = \varphi\,(t)\cdot\psi\,(\vartheta)$$

where $\varphi\,(t)$ is a function of t alone and $\psi\,(\vartheta)$ a function of ϑ alone, we get:

$$(3)\qquad \tau_0\,\frac{\varphi'}{\varphi} = -\lambda = \frac{1}{\psi}\,\frac{1}{\sin\vartheta}\,\frac{\partial}{\partial\vartheta}\left[\sin\,\vartheta\,\frac{\partial\psi}{\partial\vartheta}\right]$$

λ is a constant.

(3) divides into two equations, of which we consider

$$\lambda = -\frac{1}{\psi}\,\frac{1}{\sin\vartheta}\,\frac{\partial}{\partial\vartheta}\left[\sin\,\vartheta\,\frac{\partial\psi}{\partial\vartheta}\right].$$

This is LEGENDRE's equation, which allows only regular solutions for the proper values

$$\lambda = n\,(n+1) \qquad (\text{with } n = 0,\ 1,\ 2,\ \ldots\ldots)$$

with the appertaining proper functions ("Eigenfunktionen"), the normalized spherical harmonics of the first kind:

$$\psi_n = \sqrt{\frac{2n+1}{2}}\, P_n(x) \qquad x = \cos\vartheta .$$

After this, the second half of (3) gives:

$$\frac{\varphi'}{\varphi} = -\frac{\lambda}{\tau_0} = -\frac{n(n+1)}{\tau_0} = -\frac{1}{\tau_n} \text{ as } \tau_n = \frac{\tau_0}{n(n+1)} .$$

The solution of this equation is

$$\varphi_n = C_n e^{-\frac{t}{\tau_n}} .$$

Then the general solution of (2) is:

$$f = \sum_{n=0}^{\infty} C_n \psi_n(\cos\vartheta) e^{-\frac{t}{\tau_n}} = \sum_{n=0}^{\infty} C_n \psi_n(\cos\vartheta) e^{-\frac{n(n+1)t}{\tau_0}} .$$

By a suitable selection of the coefficients C_n it may be adjusted to any function for the time $t = 0$, and we see that, no matter what solution we start with, in the course of time we arrive at the equal distribution, all the terms except the first one being eliminated, and $P_0(x) = 1$

$f(t, \vartheta)$ is given for the time $t = 0$, for example $f(0, \vartheta) = F(\vartheta)$ where F is a given function:

$$F(\vartheta) = \sum_{n=0}^{\infty} C_n \psi_n(\cos\vartheta) .$$

In order to solve (1) with this initial condition we make a calculation of perturbation, considering α as a small value, and put:

$$f = f^0 + f^1 \tag{4}$$

where f^0 is that solution of the homogeneous equation (2) which for $t = 0$ is equal to the assumed f for $t = 0$. This means that $f^1 = 0$ at $t = 0$ for all ϑ; this makes f^0 a given known function.

As both α and f^1 are small, the products are disregarded, so that substitution of (4) in (1) gives:

$$\tau_0 \frac{\partial f^1}{\partial t} - \frac{1}{\sin\vartheta} \frac{\partial}{\partial\vartheta}\left[\sin\vartheta \frac{\partial f^1}{\partial\vartheta}\right] = -\frac{1}{2} \frac{\alpha}{\sin\vartheta}\left[\sin\vartheta \sin 2\vartheta \, f^0\right]. \tag{5}$$

(5) is a non-homogeneous differential equation in which the right side is a given function of f^0 and ϑ.

For any time we may expand $f^1(t, \vartheta)$ in terms of the proper functions of the homogeneous equation:

$$f^1(t, \vartheta) = \sum_{n=0}^{\infty} \varphi_n^1(t)\, \psi_n(\cos\vartheta). \tag{6}$$

Substitution of (6) in (5) gives for the left side:

$$\tau_0 \sum_{n=0}^{\infty} \frac{d\varphi_n^1(t)}{dt} \psi_n(\cos\vartheta) - \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left[\sin\vartheta \frac{\partial}{\partial\vartheta} \sum_{n=0}^{\infty} \varphi_n^1(t)\, \psi_n(\cos\vartheta)\right] =$$

$$\tau_0 \sum_{n=0}^{\infty} \frac{d\varphi_n^1(t)}{dt} \psi_n(\cos\vartheta) - \sum_{n=0}^{\infty} \varphi_n^1(t) \underbrace{\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left[\sin\vartheta \frac{\partial \psi_n(\cos\vartheta)}{\partial\vartheta}\right]}_{-n(n+1)\,\psi_n(\cos\vartheta)}$$

and makes the whole equation:

$$\sum_{n=0}^{\infty}\left[\tau_0 \frac{d\varphi_n^1(t)}{dt} + n(n+1)\,\varphi_n^1(t)\right]\psi_n(\cos\vartheta) =$$
$$-\frac{\alpha}{2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left[\sin\vartheta \sin 2\vartheta \, f^0\right].$$

Introduction of $x = \cos\vartheta$, as a new variable instead of ϑ, finally gives:

$$\sum_{n=0}^{\infty}\left[\tau_0 \frac{d\varphi_n^1(t)}{dt} + n(n+1)\,\varphi_n^1(t)\right]\psi_n(x) = +\alpha \frac{d}{dx}\left[x(1-x^2)\, f^0\right].$$

By multiplication of this equation with $\psi_m(x)$ and integration over x from -1 to $+1$, employing the orthogonal relation of spherical harmonics, we get

$$(7) \qquad \tau_0 \frac{d\varphi_m^1(t)}{dt} + m(m+1)\,\varphi_m^1(t) = g_m(t)$$

where $g_m(t) = +\alpha \int_{-1}^{+1} \psi_m(x) \frac{d}{dx}\left[x(1-x^2)\, f^0(t,x)\right] dx$

and $m = 0,\ 1,\ 2,\ 3,\ \ldots\infty$

In order to solve (7), using again index n instead of m, we put:

$$(8) \qquad \varphi_n^1(t) = e^{-\frac{t}{\tau_n}}\, y_n(t) = e^{-\frac{n(n+1)t}{\tau_0}}\, y_n(t).$$

By substitution we get

$$(9) \qquad \tau_0 \frac{dy_n(t)}{dt} = g_n(t)\, e^{+\frac{t}{\tau_n}}$$

(9) can be integrated directly, since y_n, like φ_n^1 and f^1, will vanish when $t = 0$. We then have

$$(10) \qquad y_n(t) = \frac{1}{\tau_0} \int_0^t g_n(t')\, e^{\frac{t'}{\tau_n}}\, dt'$$

where t' is a variable of integration, or according to (8)

$$(11) \qquad \varphi_n^1(t) = \frac{1}{\tau_0}\, e^{-\frac{t}{\tau_n}} \int_0^t g_n(t')\, e^{\frac{t'}{\tau_n}}\, dt'.$$

Substituting this expression in (6) we have f^1 as a function of t and x or of t and ϑ.

The formulae derived so far, which are quite general, will now be applied to a particular simple initial distribution.

If f for the time $t = 0$ is independent of ϑ, then $f^0 = 1$ for $t = 0$; and as $f^0 =$ constant is a solution of the homogeneous equation, $f^0 = 1$ is valid for all t.

For $g_n(t)$ we then have:

$$g_n(t) = + \alpha(t) \int_{-1}^{+1} \psi_n(x)(1 - 3x^2)\, dx.$$

But we know that

$$P_2(x) = \frac{1}{2}(3x^2 - 1) \text{ and } \psi_2 = \sqrt{\frac{5}{2}} P_2(x).$$

Therefore

$$g_n(t) = -2\sqrt{\frac{2}{5}}\alpha(t) \int_{-1}^{+1} \psi_n(x)\psi_2(x)\, dx. \tag{12}$$

So, for this simple initial condition (equal distribution of direction) all $g_n(t) = 0$, except $g_2(t)$, which is equal to $-2\sqrt{\frac{2}{5}}\alpha(t)$.

This means $\varphi_n^1(t) = 0$ for all $n \neq 2$, and

$$\varphi_2^1(t) = -\frac{1}{\tau_0} e^{-\frac{t}{\tau_2}} \int_0^t 2\sqrt{\frac{2}{5}}\alpha(t') e^{\frac{t'}{\tau_2}}\, dt'. \tag{13}$$

If now we further consider the case that a constant field is established at the time $t = 0$, α becomes constant in the time, and then we have: $\varphi_2^1(t) = -2\sqrt{\frac{2}{5}}\alpha\left(1 - e^{-\frac{t}{\tau_2}}\right)\frac{1}{6}$

$$\text{as } \int_0^t e^{\frac{t'}{\tau_2}}\, dt' = \tau_2\left(e^{\frac{t}{\tau_2}} - 1\right) \text{ and } \tau_2 = \frac{1}{6}\tau_0$$

As $f^1 = \varphi_2^1(t)\psi_2(\cos\vartheta)$ and $\psi_2(\cos\vartheta) = \frac{1}{2}\sqrt{\frac{5}{2}}(3x^2 - 1)$

we get:

$$(14) \qquad f = f^0 + f^1 = 1 - \frac{\alpha}{6}\left(1 - e^{-\frac{t}{\tau_2}}\right)(3\cos^2\vartheta - 1).$$

With $t = \infty$ formula (14) becomes the simple Maxwell distribution, as in this case we get:

$$f = 1 - \frac{\alpha}{4}\left(\cos 2\vartheta + \frac{1}{3}\right).$$

This means that, apart from values of higher orders of magnitude in α we have:

$$f \propto e^{-\frac{\alpha}{4}\left(\cos 2\vartheta + \frac{1}{3}\right)} \propto \text{const}\; e^{-\frac{\alpha}{4}\cos 2\vartheta} \propto \text{const}\; e^{-\frac{u}{kT}}$$

where

$\alpha = \dfrac{2D_0}{kT}$ and $D = D_0 \sin 2\vartheta = -\dfrac{\partial u}{\partial \vartheta}$, the last expression by integration giving

$$u = \frac{D_0}{2}\cos 2\vartheta = \frac{\alpha}{4}kT\cos 2\vartheta.$$

From (14) the relaxation time for the particles is seen to be:

$$\tau_2 = \frac{\tau_0}{6} = \frac{\xi}{6kT}.$$

If α is a function of the time $\alpha(t)$ (for example, when the field results from a sine tension), with the usual symbols for angular velocity and phase-displacement we get:

$$E = E_0 \sin(\omega t + \delta) \text{ and } D = D_0 \sin 2\vartheta \sin^2(\omega t + \delta)$$

$$D_0 \sin^2(\omega t + \delta) = D_0\left[\frac{e^{i(\omega t + \delta)} - e^{-i(\omega t + \delta)}}{2i}\right]^2 =$$

$$\frac{D_0}{2} - \frac{D_0}{2} \text{ Real part of } \left\{e^{i(2\omega t + 2\delta)}\right\} \text{ that is}$$

$$\alpha(t) = \frac{2 D_0 \sin^2(\omega t + \delta)}{kT} = \frac{D_0}{kT} - \frac{D_0}{kT} R \left\{ e^{i(2\omega t + 2\delta)} \right\}$$

Introducing this $\alpha(t)$ into the expression (13) found before for $\varphi_2^1(t)$, and remembering that:

$$f^1 = \varphi_2^1(t)\, \psi_2(\cos\vartheta)$$

we get by fairly simple calculations:

$$= f^0 + f^1 = 1 - \frac{D_0}{6kT}(3\cos^2\vartheta - 1)\left\{1 - \frac{\cos[2\omega t + 2\delta] + 2\omega\tau_2 \sin[2\omega t + 2\delta]}{1 + (2\omega\tau_2)^2} \right.$$

$$\left. - e^{-\frac{t}{\tau_2}}\left[1 - \frac{\cos[2\delta] + 2\omega\tau_2 \sin[2\delta]}{1 + (2\omega\tau_2)^2}\right]\right\}$$

$$f = 1 - \frac{D_0}{6kT}(3\cos^2\vartheta - 1)\left\{1 - \frac{1}{\sqrt{1 + (2\omega\tau_2)^2}}\cos[2\omega t + 2\delta + b] \right.$$

$$\left. - e^{-\frac{t}{\tau_2}}\left[1 - \frac{1}{\sqrt{1 + (2\omega\tau_2)^2}}\cos[2\delta + b]\right]\right\}$$

$$f = 1 - \frac{D_0}{6kT}(3\cos^2\vartheta - 1)\, G(t)$$

where $b = \text{arc tg}\,(-2\omega\tau_2)$ signifies a phase-displacement, and $G(t)$ an abbreviation for the $\{\}$ parenthesis.

We find that the distribution of direction varies with twice the frequency of the field strength. This is easy to understand, as the rotation moment (cf. Section I) depends on the square of the field strength.

For $\omega = 0$ $\delta = \frac{\pi}{2}$ we get $G(t) = 2\left(1 - e^{-\frac{t}{\tau_2}}\right)$ and f equal to expression (14) with

$$\alpha \text{ const} = \frac{2D_0}{kT}$$

For $(\omega\tau_2) >> 1$ we get

$$f = 1 - \frac{D_0}{6\,kT}(3\cos^2\vartheta - 1)\left(1 - e^{-\frac{t}{\tau_2}}\right) = 1 - \frac{\alpha}{12}(3\cos^2\vartheta - 1)\left(1 - e^{-\frac{t}{\tau_2}}\right)$$

that is, when the oscillation periods are small in proportion to the time of relaxation, the expression for the deviation from the equal distribution is exactly one half of the corresponding expression for direct current.

IV. Calculation of the Conductivity of a Suspension for a Given Time.

H. Fricke[1] has calculated the conductivity of a suspension of ellipsoids, assuming a random orientation of the ellipsoids.

As we are here interested in the orientating effect of the electric field on the conductivity of the suspension, this method must be generalized.

We consider again (see Section I) a coordinate system x, y, z fixed in the ellipsoid, where the x-axis coincides with the symmetrical axis and the z-axis is perpendicular to the direction of the applied external electric field. We further introduce the coordinate system x', y', z', having the same point of origin and z-axis as the x, y, z system and the x'-axis situated in the direction of the field.

The components of the applied field in the directions of the x-, y- and z-axes are:

$$E_x = E\cos\vartheta \qquad E_y = E\sin\vartheta \qquad E_z = 0$$

ϑ, as previously, signifying the acute angle between the symmetrical axis (x-axis) and the direction of the field (x'-axis).

For the components of the field strenght inside an

[1] H. Friche: Phys. Rev. **24**: 575, 1924.

ellipsoid calculated according to directions of the main axes (x, y, z) we have (cf. FÜRTH[1] p. 102):

$$\mathrm{E}'_x = -\frac{\partial\varphi}{\partial x} = -\alpha_1 \quad \mathrm{E}'_y = -\frac{\partial\varphi}{\partial y} = -\beta_1 \quad \mathrm{E}'_z = -\frac{\partial\varphi}{\partial z} = -\gamma_1$$

where

$$\alpha_1 = -\frac{E_x}{1+\frac{\mu-1}{4}X_0} = -\frac{E\cos\vartheta}{1+\frac{\mu-1}{4}X_0}$$

$$\beta_1 = -\frac{E_y}{1+\frac{\mu-1}{4}Y_0} = -\frac{E\sin\vartheta}{1+\frac{\mu-1}{4}Y_0}$$

$$\gamma_1 = -\frac{E_z}{1+\frac{\mu-1}{4}Z_0} = -\frac{0}{1+\frac{\mu-1}{4}Z_0} = 0$$

and

$$X_0 = 2ab^2\int_0^\infty \frac{dt}{(a^2+t)\,T(t)} \qquad Y_0 = 2ab^2\int_0^\infty \frac{dt}{(b^2+t)\,T(t)}$$

$$T(t) = \sqrt{a^2+t}\,(b^2+t)$$

the x' component of the intensity of the field inside the ellipsoid $\mathrm{E}'_{x'}$ can be calculated by means of the formula

$$A_s = A_x\cos(s,x) + A_y\cos(s,y) + A_z\cos(s,z)$$

which gives

$$(1) \quad \mathrm{E}'_{x'} = -\alpha_1\cos\vartheta - \beta_1\sin\vartheta = \frac{E\cos^2\vartheta}{1+\frac{\mu-1}{4}X_0} + \frac{E\sin^2\vartheta}{1+\frac{\mu-1}{4}Y_0}.$$

The intensity of the field outside the ellipsoids will be called $\underline{\mathrm{E}}^0$. In the immediate proximity of the ellipsoid $\underline{\mathrm{E}}^0$ differs from $\underline{E}$, but at great distances from the ellipsoids the two values are identical.

[1] R. FÜRTH: Zeit. f. Physik: **22**: 98, 1924.

The volume between the electrodes is called Ω, and the total volume of the ellipsoids Ω_1.

Let $\underline{F}_o$ express the strength of the electric field at an arbitrary point of the suspending medium. For a definite assumed distribution and orientation of the ellipsoids we now form the mean value taken over the space outside the ellipsoids, and call it

$$(2) \qquad \underline{F} = \overline{\underline{F}_o}\Big|_{\text{outside ellipsoids}}.$$

We know that it is a vector, at right angles to the electrodes, that is

$$(3) \qquad F_{x'} = F \text{ and } F_{y'} = F_{z'} = 0\,.$$

Inside the ellipsoids there is a certain intensity of electric field $\underline{F}_i$ which, strictly speaking, varies from point to point within the ellipsoids.

Considering all the ellipsoids whose symmetrical axis (x-axis) forms an angle between ϑ and $\vartheta + d\vartheta$ with the direction of the field (x'-axis), and forming the mean value of $\underline{F}_i$ taken over the volume of these ellipsoids, we get a vector $\underline{F}_i(\vartheta)$, that is:

$$(4) \qquad \underline{F}_i(\vartheta) = \overline{\underline{F}_i}\Big|_{\vartheta,\ \vartheta + d\vartheta}.$$

It is natural to assume that this mean strength of the field $\underline{F}_i(\vartheta)$ is the same as the constant field strength in a single ellipsoid placed in a suitably chosen external constant field $\underline{E}$, and whose symmetrical axis forms the angle ϑ with the direction of this field.

Now the problem is how we are to choose this field $\underline{E}$. The condition must be that the mean value of $\underline{\mathrm{E}}^0$ (see above) taken over a volume $(\Omega - \Omega_1)$ outside the ellipsoid is equal to the $\underline{F}$ found above.

It is found that if the volume Ω is large in proportion to the volume of a single ellipsoid, the mean value of $\underline{E}^0$ within this region will be approximately equal to $\underline{E}$, that is, that we have to put

$$(5) \qquad \underline{E} = \underline{F} \text{ and } \underline{E}' = \underline{F}_i(\vartheta)$$

$$(\underline{F}_i(\vartheta)\text{'s components } E'_{x'} \text{ etc.})$$

In order to get the connection between $\underline{F}$ and V (the potential between the electrodes), we note that the mean value of the true field strength taken over the entire volume Ω must be equal to $\frac{V}{l}$, where l is the distance between the plates, i. e.,

$$(6) \qquad \overline{(F_o)_{x'}}\Big|_{\text{outside ellipsoids}} (\Omega - \Omega_1) + \overline{(F_i)_{x'}}\Big|_{\text{inside ellipsoids}} \Omega_1 = \frac{V}{l}\,\Omega$$

which is equal to

$$(6') \qquad F(\Omega - \Omega_1) + \Omega_1 \int_{\vartheta=0}^{\vartheta=\pi} \big(F_i(\vartheta)\big)_{x'} N(\vartheta)\, d\vartheta = \frac{V}{l}\,\Omega$$

where $N(\vartheta)d\vartheta$ means the relative number of ellipsoids within Ω, whose symmetrical axes forms an angle between ϑ and $\vartheta + d\vartheta$ with the x'-axis, and normalized so that

$$\int_0^\pi N(\vartheta)\, d\vartheta = 1$$

By employing (1) and (5) we get, instead of (6′):

$$(7) \quad (\Omega - \Omega_1) F + \Omega_1 F \int_0^\pi \left\{ \frac{\cos^2\vartheta}{1 + \frac{\mu - 1}{4} X_0} + \frac{\sin^2\vartheta}{1 + \frac{\mu - 1}{4} Y_0} \right\} N(\vartheta)\, d\vartheta = \frac{V}{l}\,\Omega$$

This relation replaces Fricke's[1] equation 7, p. 580.

[1] H. Friche: Phys. Rev. **24**: 575, 1924.

When ϱ means the volume concentration $\varrho = \frac{\Omega_1}{\Omega}$, we get by dividing (7) by Ω:

$$(8) \qquad F(1-\varrho) + \varrho F \int_0^{\pi} \left\{ \frac{\cos^2\vartheta}{A} + \frac{\sin^2\vartheta}{B} \right\} N(\vartheta)\, d\vartheta = \frac{V}{l}$$

where

$$(8') \qquad A = 1 + \frac{\mu - 1}{4} X_0 \qquad B = \frac{\mu - 1}{4} Y_0 .$$

When we know the mean value of the field strength, we also know the mean value of the density of the current $\underline{u}$, as it applies to every point that

$$\underline{u} = \sigma \underline{\mathrm{E}}$$

so that the mean value of $\underline{u}$ through a region of space where σ is constant is equal to σ times the mean value of the field strength.

So, for the mean value of $\underline{u}$ outside the ellipsoids, we get

$$\underline{u}^0 = \overline{\underline{u}_0}\Big|_{\text{outside ellipsoids}} = \sigma^0 \overline{\underline{F}_0}\Big|_{\text{outside ellipsoids}} = \sigma^0 \underline{F} .$$

$\underline{u}^0$ is perpendicular to the electrodes, and equal to $\sigma^0 F$.

The mean value of the x' component of the current-density inside the ellipsoids, the direction of whose symmetrical axes is between ϑ and $\vartheta + d\vartheta$, is likewise:

$$\left(u_i(\vartheta)\right)_{x'} = \sigma \left(F_i(\vartheta)\right)_{x'} = \sigma\, \mathrm{E}'_{x'} = \sigma \underline{E} \left(\frac{\cos^2\vartheta}{A} + \frac{\sin^2\vartheta}{B} \right)$$
$$= \sigma F \left(\frac{\cos^2\vartheta}{A} + \frac{\sin^2\vartheta}{B} \right)$$

Hence the mean value of the x' component of the current-density taken through the entire region of space Ω becomes equal to:

$$(9)\quad \sigma^0 \underline{F}(\Omega-\Omega_1)+\Omega_1\int_0^{\pi}\sigma F\left(\frac{\cos^2\vartheta}{A}+\frac{\sin^2\vartheta}{B}\right)N(\vartheta)\,d\vartheta=\overline{u}_{x'}\,\Omega$$

$$(9')\quad \sigma^0 F(1-\varrho)+\varrho\sigma F\int_0^{\pi}\left(\frac{\cos^2\vartheta}{A}+\frac{\sin^2\vartheta}{B}\right)N(\vartheta)\,d\vartheta=\overline{u}_{x'}.$$

The conductivity of the suspension σ_m is defined by the equation:

$$(10)\qquad \overline{u}_{x'}=\sigma_m\frac{V}{l}.$$

We put

$$I=\int_0^{\pi}\left(\frac{\cos^2\vartheta}{A}+\frac{\sin^2\vartheta}{B}\right)N(\vartheta)\,d\vartheta$$

and instead of (8) and (9′) we get

$$(11)\qquad F(1-\varrho)+\varrho FI=\frac{V}{l}$$

$$(12)\qquad \sigma^0 F(1-\varrho)+\sigma\varrho FI=\sigma_m\frac{V}{l},$$ which gives

$$(13)\qquad \sigma_m=\frac{\sigma^0+\varrho(\sigma I-\sigma^0)}{1+\varrho(I-1)}.$$

Now, obviously, the relative number of ellipsoids, the direction of whose symmetrical axes falls between ϑ and $\vartheta+d\vartheta$, is proportional to

$$f(\vartheta)\sin\vartheta\,d\vartheta$$

that is $$N(\vartheta)\,d\vartheta=C\,f(\vartheta)\sin\vartheta\,d\vartheta$$

where the constant C can be determined by the normalizing equation:

$$\int_0^{\pi}N(\vartheta)\,d\vartheta=\int_0^{\pi}C\,f(\vartheta)\sin\vartheta\,d\vartheta=1.$$

The function $f(\vartheta)$ is normalized so that the integral, being extended over all directions, is equal to 4π, since we started with an equal distribution; and this integral is constant in the course of time, that is:

$$4\pi = \int_{\text{sphere of unit}} f(\vartheta)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} f(\vartheta)\, d\varphi \sin\vartheta\, d\vartheta = 2\pi\int_0^{\pi} f(\vartheta)\sin\vartheta\, d\vartheta.$$

This means that $C = \frac{1}{2}$ or

$$N(\vartheta)\, d\vartheta = \frac{1}{2} f(\vartheta)\sin\vartheta\, d\vartheta$$

and thus we get

$$I = \int_0^{\pi}\left(\frac{\cos^2\vartheta}{A} + \frac{\sin^2\vartheta}{B}\right)\frac{1}{2} f(\vartheta)\sin\vartheta\, d\vartheta.$$

Substitution of $x = \cos\vartheta$, as a new variable of integration gives:

$$I = -\int_1^{-1}\left(\frac{x^2}{A} + \frac{1-x^2}{B}\right)\frac{1}{2} f(x)\, dx.$$

For equal distribution of direction it holds that $f = f^0 = 1$; the corresponding I is called I_0

$$I_0 = \frac{1}{2}\int_{-1}^{+1}\left(\frac{x^2}{A} + \frac{1-x^2}{B}\right) dx = \frac{1}{3}\left(\frac{1}{A} + \frac{2}{B}\right).$$

This value introduced in (13) then gives σ_m for a random distribution of direction, in agreement with H. Fricke[1].

For α independent of the time $t\,(\omega \to 0)$ we found above (p. 18):

$$f = 1 - \frac{\alpha}{6}(3\cos^2\vartheta - 1)\left(1 - e^{-\frac{t}{\tau_2}}\right)$$

By again introducing $x = \cos\vartheta$ as a variable of integration, we find for the corresponding I

[1] H. Fricke: Physical Review. **24**: 575, 1924.

$$I_{\alpha \text{ const.}} = I_0 - \frac{\alpha}{4}\left(1 - e^{-\frac{t}{\tau_2}}\right)\int_{-1}^{+1}\left(\frac{x^2}{A} + \frac{1-x^2}{B}\right)\left(x^2 - \frac{1}{3}\right)dx$$

$$= I_0 - \frac{2\alpha}{45}\left(1 - e^{-\frac{t}{\tau_2}}\right)\left(\frac{1}{A} - \frac{1}{B}\right).$$

We see that I decreases from the value I_0 at the original state of equal distribution to the value $I_{\alpha \text{ constant}}$.

The time of relaxation — i. e. the time that passes before the conductivity assumes its new constant value—is $\tau_2 = \frac{\tau_0}{6}$, as stated already on p. 18.

In the case of $\sigma_0 >> \sigma I$, we see from (13) that a decrease of I means an increase of the conductivity in the course of time.

If on the other hand, σ is so great in proportion to σ_0 that σ_0 can be considered small in proportion to σI, a decrease of I means a decrease in the conductivity of the suspension.

For α dependent on the time, $\alpha(t)$, we found (p. 19)

$$f = 1 - \frac{\alpha}{4}\left(x^2 - \frac{1}{3}\right)G(t).$$

So the corresponding I becomes

$$I_{\alpha (t)} = I_0 - \frac{\alpha}{8}G(t)\int_{-1}^{+1}\left(\frac{x^2}{A} + \frac{1-x^2}{B}\right)\left(x^2 - \frac{1}{3}\right)dx$$

$$= I_0 - \frac{\alpha}{45}\left\{\frac{1}{A} - \frac{1}{B}\right\}\left\{1 - \frac{1}{\sqrt{1+(2\omega\tau_2)^2}}\cos\left[2\omega t + 2\delta + b\right]\right.$$

$$\left. - e^{-\frac{t}{\tau_2}}\left[1 - \frac{1}{\sqrt{1+(2\omega\tau_2)^2}}\cos\left[2\delta + b\right]\right]\right\}.$$

For $(\omega\tau_2) >> 1$ we get:

$$I_{\alpha (t)} = I_0 - \frac{\alpha}{45}\left(\frac{1}{A} - \frac{1}{B}\right)\left(1 - e^{-\frac{t}{\tau_2}}\right)$$

where the term containing α is half as great as the corresponding term in the expression for $I_{\alpha\,\text{const.}}$

V. Numerical Examples of the Preceding Calculations.

The effect of orientation depends especially on the term characteristic of our theory

$$\alpha = \frac{2\,D_0}{kT}\left(\sim \frac{\text{field energy per ellipsoid}}{kT}\right)$$

that is, of $\varepsilon_0, E, \mu = \frac{\sigma}{\sigma^0}, a, b$ and T, (X_0 and Y_0 depend only on $\varkappa = \frac{b}{a}$, λ also on a; $I_0 = \frac{1}{3}\left(\frac{1}{A}+\frac{2}{B}\right)$ and $\left(\frac{1}{A}-\frac{1}{B}\right)$ depend on μ and $\varkappa$), besides on the volume concentration ϱ (formula (13), Section IV.)

The duration of the effect is determined by the time of relaxation

$$\tau_2 = \frac{\xi}{6\,kT}$$

which depends on η, besides on a, b and T.

From the formula for the rotation moment D (see Section I) it is evident that we have to distinguish between two main cases:

I. $\mu >> 1$ and II. $\mu << 1$, where we may obtain considerable effects in the first case, and only insignificant ones in the second, as the value of the denominator in the expression for D chiefly depends on μ.

For the sake of simplicity, examples are calculated for rod-shaped particles (elongated ellipsoids of revolution) only.

I. $\mu >> 1$ $\quad$ $\sigma = 1\times10^{-4}$ rec. Ohm. cm^{-1} $\quad$ $a = 4\times10^{-4}$ cm

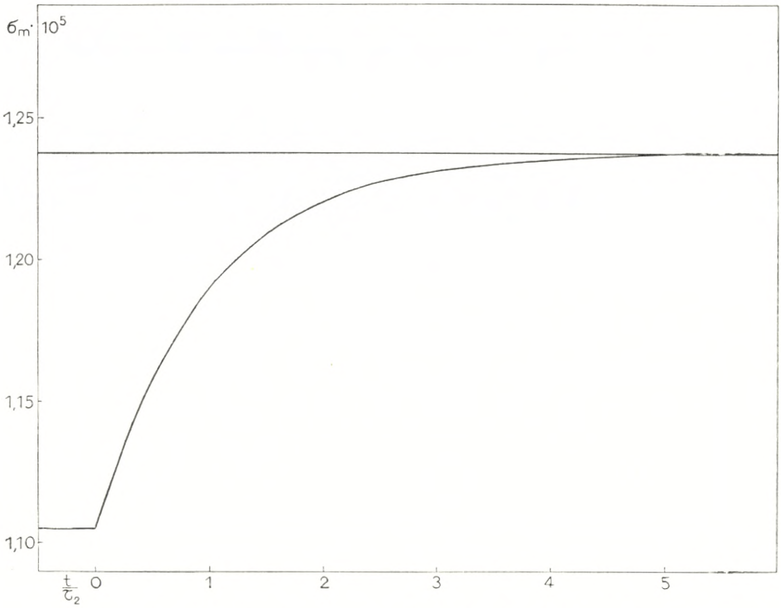
$\sigma^0 = 1\times10^{-6}$ rec. Ohm. cm^{-1} $\quad$ $b = 0{,}5\times10^{-4}$ cm

$\mu = \frac{\sigma}{\sigma^0} = 100$ $\quad$ $\varkappa = \frac{b}{a} = \frac{1}{8}$ $\quad$ $T = 291$ Kelvin (18°C.)

Sine current 1000 Hertz. E_0 = Maximal tension/cm = 2 volts/cm = $\frac{2}{300}$ electrostatic unit/cm.

$$\alpha = -2{,}4284 \qquad I_{\alpha\ \mathrm{const.}} = 0{,}26358 + 0{,}03939 \cdot \left(1 - e^{-\frac{t}{\tau_2}}\right)$$

For $\eta = 0{,}05$ Poise we have $\tau_2 = 159{,}4$ seconds.



By calculating $I_{\alpha\ \mathrm{const.}}$ and substituting it in formula (13), Section IV, we get the conductivity for various lengths of time.

Fig. 1 gives a graphic presentation of the orientation effect when the volume concentration of the suspension $\varrho = 0{,}3$, that is, when it amounts to

$$\frac{\varrho}{\frac{4}{3}ab^2\pi} = \text{about } 7\times 10 \text{ particles/c.c.}$$

II. $\mu << 1$ $\qquad \sigma = 1\times 10^{-6}$ rec. Ohm. cm^{-1} $\qquad a = 4\times 10^{-4}$ cm

$\sigma^0 = 1\times 10^{-4}$ rec. Ohm. cm^{-1} $\qquad b = 0{,}5\times 10^{-4}$ cm

$\mu = \frac{1}{100}$ $\qquad \varkappa = \frac{1}{8}$ $\qquad T = 291$ Kelvin (18°C.)

Sine current 1000 Hertz E_0 = Maximal tension/cm = $\frac{2}{300}$ electrostatic units/cm.

$\alpha = -0{,}03226 \quad I_{\alpha\,(t)} = 1{,}6502 - 0{,}0007\left(1 - e^{-\frac{t}{\tau_2}}\right)$. For $\eta = 0{,}05$ Poise we have $\tau_2 = 159{,}4$ seconds.

Here the orientation effect becomes insignificant, as the conductivity increases only from $5{,}8987 \times 10^{-5}$ to $5{,}8996 \times 10^{-5}$ rec. Ohm. cm^{-1} that is, $0{,}0009 \times 10^{-5}$ rec. Ohm. cm^{-1} from the time 0 to the time ∞.

The slight rotation moment in case II explains why Freundlich and Abramson[1] and Abramson[2] in cataphoretic experiments found no orientation of the red blood corpuscles of horse (which may approximately be considered oblate ellipsoids of revolution), either when situated singly or in rouleaux ("piles of coins").

The conductivity of suspensions of blood corpuscles, with special reference to the phenomenon demonstrated by H. Fricke and S. Morse[3] and by Mc.Clendon[4]—that the conductivity of blood changes in the course of time when the stirring up or agitation movement have ceased—will be discussed in a subsequent paper.

As D depends on a^3, the orientation effect will be very slight for very small particles, even in case I. ($\mu >> 1$).

In the experiment performed by Kruyt[5] with a colloid solution of vanadium pentoxide ($a = 0{,}5$ micron) it must be assumed, therefore, that the particles are orientated through cataphoretic liquid currents in the very shallow observation chamber (as, *ceteris paribus*, a decrease of a from 4×10^{-4} to $0{,}5 \times 10^{-4}$ will reduce D by $64/0{,}125 = 512$ times).

[1] H. Freundlich and H. A. Abramson: Zeit. f. phys. Chem. **133**: 51, 1928.
[2] H. A. Abramson: Proceed. Soc. Exp. Biol. and Med. **26**: 147, 1928—29.
[3] H. Fricke and S. Morse: Journ. of Gen. Physiol. **9**: 153, 1926.
[4] McClendon: Journ. Biol. Chem. **68**: 653, 1926.
[5] Kruyt: Kolloidz. **19**: 161, 1916.

The different views held by French physicists (Cotton and Mouton, Meslin and Chaudier) and by German colloid chemists (Freundlich and collaborators, Kruyt) as to the cause of the orientation of the particles by "electrical double refraction" in suspensions have been discussed by C. E. Marshall.[1]

Summary.

In a suspension of homogeneous ellipsoids it is assumed that a change in the electrical conductivity takes place in the course of time, owing to an orientation of the ellipsoids produced by that moment of rotation with which the electric field acts upon the ellipsoids.

In the present paper this effect is calculated for ellipsoids of revolution.

For the electric rotation moment acting on an ellipsoid of revolution I have used R. Fürth's formula, which is valid if quasi-stationary conditions be assumed.

For the antagonistic moment of friction I have used Edwardes' formula.

By means of a method analogous to the one employed by P. Debye for polar molecules, a function of distribution is calculated, which describes the orientation of the ellipsoids under the influence of an arbitrary electric field and of the Brownian molecular movements.

Finally, in agreement with H. Fricke, a formula is calculated for the conductivity of the suspension when the ellipsoids of revolution have a given distribution of direction, so that it is possible to estimate the conductivity at a given time and follow its change in the course of time.

For a quite random distribution of the ellipsoids the

[1] C. E. Marshall: Transact. of the Faraday Soc. **26**: 173, 1930.

formula for the conductivity is identical with the one worked out by H. Fricke.

For a constant field the conductivity changes with the time, and after some length of time (relaxation time) it assumes a new constant value that differs somewhat from the one that corresponds to equal distribution of direction.

For alternating voltage, the periods of which are small in proportion to the time of relaxation, the change in conductivity (orientation effect) is found to be in simple relation to the change observed under constant voltage.

For suspensions of ellipsoids whose conductivity is many times greater than that of the medium, considerable effects may be obtained. If the conductivity of the suspending medium is many times greater than that of the particles, only a slight effect is obtained.

When the particles are very small, the effect is insignificant in every instance.

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